IYSE 6420 Fall 2020 Homework4

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1. Simple Metropolis: Normal Precision – Gamma.

**Suppose was observed from the population distributed as and one wishes to estimate the parameter .**

**(Here is the reciprocal of the variance and is called the precision parameter). Suppose the analyst believes that the prior on is .**

**Using Metropolis algorithm, approximate the posterior distribution and the Bayes’ estimator of . As the proposal distribution, use gamma with parameters selected to ensure efficacy of the sampling (this may require some experimenting).**

Likelihood

Log-likelihood

When , MLE

If prior is , posterior

is gamma

Bayes estimator for

When , posterior is gamma , which is also exponential

The bayes estimator is the mean of posterior

Proposal =

Use we get bayes estimator = 0.3331, which is close to

Code:

|  |
| --- |
| close all  clear all    rand('seed',1);  randn('seed',1);  x = -2; %data  theta = 0.5; % initial value  thetas =[theta]; %save all thetas.  %    tic  for i = 1:100000  theta\_prop = randn + x; %N(x,1).  %--------------------------------------------------------------------------  r = (theta\_prop^(-0.5)\*exp(-1.85\*theta))/(theta^(-0.5)\*exp(-1.85\*theta\_prop));  %--------------------------------------------------------------------------  rho = min(r ,1);  if (rand < rho)  theta = theta\_prop;  end  thetas = [thetas theta];  end  toc  %Burn in 500  thetas = thetas(500:end);  figure(1)  hist(thetas, 50)  mean(thetas)  var(thetas) |

2. Normal-Cauchy by Gibbs.

Assume that is a sample from distribution, and that the prior on θ is Cauchy

Even though the likelihood for simplifies by sufficiency arguments to a likelihood of , a closed form for the posterior is impossible and numerical integration is required.

The approximation of the posterior is possible by Gibbs sampler as well. Cauchy distribution can be represented as a scale-mixture of normals:

that is

The full conditionals can be derived from the product of the densities for the likelihood and priors

(a) Show that full conditionals are normal and exponential

Let

So

From joint with respect to

So

b) Jeremy models the score on his IQ tests as with He places Cauchy

prior on .

In 10 random IQ tests Jeremy scores . The

average score is 103.5, which is the frequentist estimator of . Using Gibbs sampler described in (a) approximate the posterior mean and variance. Approximate 95% equi-tailed credible set by sample quantiles.

|  |
| --- |
| clear all  close all force  randn('state',4);    data = [100,106,110,97,90,112,120,95,96,109];  yhat = 103.5;  %  lendata=length(data);  sumdata=sum(data);    sigma2 = 90;  tau2 = 120;  mu = 110;  %  theta = 0;  thetas =[theta];  lambda = gamma(0.5);  lambdas=[lambda];  burn =1000;  ntotal = 10000 + burn;  tic  for i = 1: ntotal  theta = (tau2/(tau2 + lambda \* sigma2) \* yhat + ...  lambda \* sigma2/(tau2 + lambda \* sigma2) \* mu) + ...  sqrt(tau2 \* sigma2/(tau2 + lambda \*sigma2)) \* randn;  lambda = exprnd( 1/((tau2 + (theta - mu)^2)/(2\*tau2)));  thetas =[thetas theta];  lambdas =[lambdas lambda];  end  toc  %  mean(thetas(burn+1:end))  var(thetas(burn+1:end))  hist(thetas(burn+1:end), 40)  prctile(thetas(burn+1:end), 2.5)  prctile(thetas(burn+1:end), 97.5) |

Mean: 106.1853

Var: 55.3554

95% creditable set: (90.29, 120.08)